Performance Comparison of OT-MRC and MEC-GSC with dual branches over Correlated Nakagami-m Fading Channels

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***Abstract*— In this paper we investigate the performance of output threshold maximum ratio combining (OT-MRC) and minimum estimation and combining generalized selection combining (MEC-GSC) scheme with dual branches over correlated Nakagami-m fading channels. The OT-MRC and MEC-GSC schemes select the minimum number of diversity branches such that their combined received signal-to-noise ratio (SNR) is above a certain pre-determined target SNR that is set to meet the required quality of service (QoS). Based on the joint probability density function (PDF) and moment generating function (MGF) of the combined received SNR, the performance comparisons of dual OT-MRC and MEC-GSC over correlated Nakagami-m fading channels are presented.**

***Index Terms*— Diversity, output threshold maximum ratio combining, minimum estimation and combining generalized selection that MEC-GSC schem can attain the fading channel**

I. Introduction

There has been much interest in diversity combining schemes as a good way of mitigating the deleterious effect of wireless fading channels without using the expensive resources such as frequency band or/and the power of the transmitted signal [1]. The maximal ratio combining (MRC), which is known as the optimal diversity schemes, processes the signal in order to provide the maximum combined output signal-to-noise (SNR). As suboptimal schemes, the selection combining (SC) selects the strongest signal for the detection among the available diversity branches. The mixture of MRC and SC, called generalized SC (GSC), combines some selected branches in order of the signal strength in a way of MRC. All these combining schemes require the channel estimation of all available paths, which leads to the high complexity in the implementation. In [2], [3], the low complexity diversity systems named as output threshold MRC (OT-MRC) and minimum estimation and combining GSC (MEC-GSC) were presented, where each scheme is designed to lower the complexity while they meet required performance.

The most popular assumption in studying the performance of diversity schemes is identically and independently distributed branches. However, independent fading is not always realizable in practice [4]. Especially for small-size mobile terminals equipped with multiple antennas, they may have insufficient antenna spacing to obtain independent fading in each branch. Additionally, they might not have enough space to place more than two antennas due to the size requirements. Under these restrictions and limitations, investigating the performance of dual branch diversity schemes in a correlated fading channel environment has some practical meanings.

In this paper we consider the dual branch OT-MRC and MEC-GSC schemes, which may be better applicable to the mobile terminal owing to their low complexity[1], over correlated Nakagami-m fading channels and compare their performance in average output SNR and average error rate to see the effect of the correlation.

II. Statistics of OT-MRC and MEC-GSC with dual branches

A. PDF

1) OT-MRC: Let γ_1 and γ_2 denotes the instantaneous SNR of the first and second branch, respectively. If the channel statistics is known, the joint PDF of Γ and L_c for OT-MRC with dual branches is given, based on its mode of operation, as [5, eq. (5)]

$$f_{\gamma_{\mathrm{OM}}}(\gamma, L_c) = \begin{cases} f_1(\gamma), & \gamma_T \le \gamma < \infty, L_c = 1, \\ \int_0^{\gamma_T} f(\gamma_1, \gamma-\gamma_1)d\gamma_1, & \gamma_T \le \gamma < \infty, L_c = 2, \\ f_{\mathrm{MRC}}(\gamma), & 0 \le \gamma < \gamma_T, L_c = 2, \end{cases} \quad (1)$$

where $f_1(\gamma)$ is the PDF of γ_1, $f_{\mathrm{MRC}}(\gamma)$ is the PDF of the output SNR of MRC over the given channels, and $f(\gamma_1,\gamma_2)$ is the joint PDF of γ_1 and γ_2.

When γ_1 and γ_2 are jointly correlated Nakagami-m random variables, the joint PDF of γ_1 and γ_2 is given by [6, eq. (126)]

$$f(\gamma_1,\gamma_2) = \left(\frac{1}{\sqrt{\sigma_1\sigma_2}}\right)^{m+1} \frac{(\gamma_1\gamma_2)^{\frac{m-1}{2}}}{\Gamma(m)\rho^{\frac{m-1}{2}}(1-\rho)} \times \exp\left[-\frac{1}{1-\rho}\left(\frac{\gamma_1}{\sigma_1}+\frac{\gamma_2}{\sigma_2}\right)\right] I_{m-1}\left(\frac{2\sqrt{\rho\gamma_1\gamma_2}}{\sqrt{\sigma_1\sigma_2}(1-\rho)}\right), \quad (2)$$

[1]The low complexity may be interpreted as the low power consumption.

where $\sigma_1 \triangleq \overline{\gamma}_1/m$, $\sigma_2 \triangleq \overline{\gamma}_2/m$, $\overline{\gamma}_l$ is the average SNR of the lth branch (l=1,2), m is the Nakagami fading parameter ($m \geq 0.5$), ρ is the correlation coefficient between γ_1 and γ_2, $\Gamma(\cdot)$ is the gamma function, and $I_v(\cdot)$ is the modified Bessel function of the first kind of order v.

Substituting of (2) into (1) and using the PDF of Nakagami-m fading channel model [6, eq. (3)] and the PDF of MRC output over two correlated Nakagami-m branches given in [6, eq. (142)], we can write the joint PDF of Γ and L_c as

$$\begin{aligned} & f_{\gamma_{\mathrm{OM}}}(\gamma, L_c) \\ &= \begin{cases} e^{-\frac{\gamma}{\sigma_1}} \dfrac{\gamma^{m-1}}{\Gamma(m)\sigma_1{}^m}, & \gamma_T \leq \gamma < \infty, L_c = 1, \\ f^*(\gamma), & \gamma_T \leq \gamma < \infty, L_c = 2, \\ \dfrac{\sqrt{\pi} e^{-\alpha\gamma} \xi^m}{\Gamma(m)} \left(\dfrac{\gamma}{2\beta}\right)^{m-\frac{1}{2}} I_{m-\frac{1}{2}}(\beta\gamma), & 0 \leq \gamma < \gamma_T, L_c = 2, \end{cases} \end{aligned} \tag{3}$$

where

$$\begin{aligned} & \alpha \triangleq \frac{\sigma_1 + \sigma_2}{2\sigma_1\sigma_2(1-\rho)}, \quad \beta^2 \triangleq \frac{(\sigma_1 - \sigma_2)^2 + 4\sigma_1\sigma_2\rho}{4\sigma_1^2\sigma_2^2(1-\rho)^2}, \\ & \xi \triangleq \frac{1}{\sigma_1\sigma_2(1-\rho)}. \end{aligned} \tag{4}$$

In (3) $f^*(\gamma)$ can be derived, using the series expansion of the Bessel function [7, eq. (8.445)] and the binomial expansion, as

$$\begin{aligned} f^*(\gamma) &= \int_0^{\gamma_T} f(\gamma_1, \gamma - \gamma_1) d\gamma_1 \\ &= \sum_{k=0}^{\infty} \sum_{i=0}^{m+k-1} \binom{m+k-1}{i} \sum_{j=0}^{2i} \binom{2i}{j} \\ &\times \frac{\rho^k (-1)^{3i-j} \xi^{m+k}}{k!\Gamma(m+k)\Gamma(m)(1-\rho)^k \chi^{j+1}} \left(\frac{\gamma}{2}\right)^{2m+2k-j-2} \\ &\times e^{-\sigma_1\xi\gamma} \gamma(j+1, \chi\gamma_T), \end{aligned} \tag{5}$$

where

$$\chi \triangleq \frac{\sigma_2 - \sigma_1}{\sigma_1\sigma_2(1-\rho)}. \tag{6}$$

For identically distributed fading channels, i.e., $\sigma_1 = \sigma_2 = \sigma$, (5) is simplified with the help of the L'Hospital's rule as

$$\begin{aligned} f^*(\gamma) &= \sum_{k=0}^{\infty} \frac{\rho^k \xi^{m+k} e^{-\sigma\xi\gamma} \gamma^{2m+2k-1}}{k!\Gamma(m+k)\Gamma(m)(1-\rho)^k} \\ &\times B\left(\frac{\gamma_T}{\gamma}, m+k, m+k\right), \end{aligned} \tag{7}$$

where $B(x,p,q)$is the incomplete beta function [7, eq. (8.391)].

As a special case, for independent identically distributed fading channels, i.e., $\rho = 0$, (7) can be further reduced to

$$f^*(\gamma) = e^{-\frac{\gamma}{\sigma}} \frac{\gamma^{2m-1}}{\sigma^{2m}[\Gamma(m)]^2} B\left(\frac{\gamma_T}{\gamma}, m, m\right), \tag{8}$$

which agrees with [5, eq. (9)].

2) MEC-GSC: Based on [3], we can write the PDF of the MEC-GSC output SNR, γ_{MEC}, as

(i) for $\gamma_T \leq \gamma < \infty$, and $L_c = 1$,

$$f_{\gamma_{\mathrm{MEC}}}(\gamma, L_c) = f_1(\gamma) + \int_0^{\gamma_T} f(\gamma_1, \gamma) d\gamma_1 \tag{9}$$

(ii) for $\gamma_T \leq \gamma < 2\gamma_T$, and $L_c = 2$,

$$f_{\gamma_{\mathrm{MEC}}}(\gamma, L_c) = \int_0^{\frac{\gamma}{2}} f(\gamma - \gamma_2, \gamma_2) d\gamma_2 - \int_{\gamma_T}^{\gamma} f(\gamma_1, \gamma - \gamma_1) d\gamma_1, \tag{10}$$

(iii) for $0 \leq \gamma < \gamma_T$, and $L_c = 2$,

$$f_{\gamma_{\mathrm{MEC}}}(\gamma, L_c) = f_{\mathrm{MRC}}(\gamma), \tag{11}$$

where $f_1(\gamma)$ is the PDF of γ_1 and $f_{\mathrm{MRC}}(\gamma)$ is the PDF of the output SNR of MRC over the given channels. Substituting (2) into (9)-(11) , we have the PDF given as

(i) For $\gamma_T \leq \gamma < \infty$, and $L_c = 1$,

$$\begin{aligned} f_{\gamma_{\mathrm{MEC}}}(\gamma, L_c) &= \frac{\gamma^{m-1} e^{-\frac{\gamma}{\sigma_1}}}{\Gamma(m)\sigma_1{}^m} \\ &+ \frac{\gamma^{m-1} e^{-\frac{\gamma}{\sigma_2}}}{\Gamma(m)\sigma_2{}^m} \left[1 - Q_m(\sqrt{2\sigma_1\xi\rho\gamma}, \sqrt{2\sigma_2\xi\gamma_T})\right] \end{aligned} \tag{12}$$

(ii) for $\gamma_T \leq \gamma < \infty$, and $L_c = 2$,

$$f_{\gamma_{\mathrm{MEC}}}(\gamma, L_c) = 2f^*(\gamma) - \frac{\sqrt{\pi} e^{-\alpha\gamma} \xi^m}{\Gamma(m)} \left(\frac{\gamma}{2\beta}\right)^{m-\frac{1}{2}} I_{m-\frac{1}{2}}(\beta\gamma) \tag{13}$$

(iii) for $0 \leq \gamma < \gamma_T$ and $L_c = 2$,

$$f_{\gamma_{\mathrm{MEC}}}(\gamma, L_c) = \frac{\sqrt{\pi} e^{-\alpha\gamma} \xi^m}{\Gamma(m)} \left(\frac{\gamma}{2\beta}\right)^{m-\frac{1}{2}} I_{m-\frac{1}{2}}(\beta\gamma) \tag{14}$$

where $f^*(\gamma)$ is defined in (5) and $Q_m(\cdot,\cdot)$ is the generalized (mth order) Marcum Q function [8].

B. MGF

If the PDF, $f(\gamma, L_c)$, is known, the MGF for dual branch diversity schemes is easily derived by calculating

$$\mathcal{M}(s) = \int_0^{\infty} e^{s\gamma} \sum_{L_c=1}^{2} f(\gamma, L_c) d\gamma. \tag{15}$$

1) OT-MRC: Substituting (3) into (15), we obtain the MGF for dual OT-MRC as

$$\begin{aligned} \mathcal{M}_{\mathrm{OM}}(s) &= \int_{\gamma_T}^{\infty} e^{s\gamma} e^{-\frac{\gamma}{\sigma_1}} \frac{\gamma^{m-1}}{\Gamma(m)\sigma_1{}^m} d\gamma + \int_{\gamma_T}^{\infty} e^{s\gamma} f^*(\gamma) d\gamma \\ &+ \int_0^{\gamma_T} e^{s\gamma} \frac{\sqrt{\pi} e^{-\alpha\gamma} \xi^m}{\Gamma(m)} \left(\frac{\gamma}{2\beta}\right)^{m-\frac{1}{2}} I_{m-\frac{1}{2}}(\beta\gamma) d\gamma. \end{aligned}$$

Using the identity for the modified Bessel function of the first kind given in [7, eq. (8.467)], we can express (16) as

$$\begin{aligned}\mathcal{M}_{\text{OM}}(s) &= \frac{\Gamma(m,(\frac{1}{\sigma_1}-s)\gamma_T)}{\Gamma(m)\sigma_1^m(\frac{1}{\sigma_1}-s)^m} + \mathcal{M}^*(s)\\ &+ \sum_{k=0}^{m-1}\frac{\Gamma(m+k)\xi^m}{k!\Gamma(m)\Gamma(m-k)(2\beta)^{m+k}}\\ &\times\Big\{\tfrac{(-1)^k}{(\alpha-\beta-s)^{m-k}}\gamma(m-k,(\alpha-\beta-s)\gamma_T)\\ &+\tfrac{(-1)^m}{(\alpha+\beta-s)^{m-k}}\gamma(m-k,(\alpha+\beta-s)\gamma_T)\Big\}\end{aligned} \tag{16}$$

where $\gamma(m,x)$ and $\Gamma(m,x)$ are the lower and upper incomplete gamma functions, respectively, defined in [7, eq. (8.350.1) and (8.350.2)] and

$$\begin{aligned}\mathcal{M}^*(s) &= \sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\binom{m+k-1}{i}\frac{\rho^k\xi^{m+k}(-1)^i(1-\rho)^{-k}}{k!\Gamma(m+k)\Gamma(m)\chi^{m+k+i}}\\ &\times\frac{\gamma(m+k+i,\chi\gamma_T)}{(\sigma_1\xi-s)^{m+k-i}}\Gamma(m+k-i,\gamma_T(\sigma_1\xi-s)).\end{aligned} \tag{17}$$

Notice that, when $\sigma_1=\sigma_2=\sigma$, (17) shrinks to

$$\begin{aligned}\mathcal{M}^*(s) &= \sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\frac{\rho^k\xi^{m+k}\gamma_T^{m+k+i}(-1)^i}{k!i!(m+k+i)\Gamma(m)\Gamma(m+k-i)(1-\rho)^k}\\ &\times(\sigma\xi-s)^{i-m-k}\Gamma(m+k-i,\gamma_T(\sigma\xi-s)),\end{aligned} \tag{18}$$

which can be further simplified, when $\rho=0$, as

$$\begin{aligned}\mathcal{M}^*(s) &= \sum_{i=0}^{m-1}\frac{\xi^m\gamma_T^{m+i}(-1)^i}{i!(m+i)\Gamma(m)\Gamma(m-i)}\\ &\times (\sigma\xi-s)^{i-m}\Gamma(m-i,\gamma_T(\sigma\xi-s)).\end{aligned} \tag{19}$$

2) MEC-GSC: Substituting (12) - (14) into (15) and adopting the expression for Marcum-Q function in [9, eq. (8)] the MGF of γ_{MG} can be written as

$$\begin{aligned}&\mathcal{M}_{\text{MEC}}(s) = \textstyle\sum_{k=0}^{m-1}\frac{\Gamma(m+k)\xi^m}{k!\Gamma(m)\Gamma(m-k)(2\beta)^{m+k}}\\ &\times\Big\{\tfrac{(-1)^k}{(\alpha-\beta-s)^{m-k}}\Big(2\gamma(m-k,(\alpha-\beta-s)\gamma_T)\\ &-\gamma(m-k,2(\alpha-\beta-s)\gamma_T)\Big)+\tfrac{(-1)^m}{(\alpha+\beta-s)^{m-k}}\\ &\times\Big(2\gamma(m-k,(\alpha+\beta-s)\gamma_T)\\ &-\gamma(m-k,2(\alpha+\beta-s)\gamma_T)\Big)\Big\}\\ &+2\mathcal{M}^{**}(s)+\tfrac{\Gamma(m,(1-\sigma_1 s)\frac{\gamma_T}{\sigma_1})}{\Gamma(m)(1-\sigma_1 s)^m}+\tfrac{\Gamma(m,(1-\sigma_2 s)\frac{\gamma_T}{\sigma_2})}{\Gamma(m)(1-\sigma_2 s)^m}\\ &-\tfrac{e^{\sigma_2\xi\gamma_T}}{\Gamma(m)\sigma_2^m}\textstyle\sum_{k=0}^{\infty}\sum_{n=0}^{m+k-1}\frac{1}{k!n!}(\sigma_1\xi\rho)^k(\sigma_2\xi\gamma_T)^n\\ &\times(\tfrac{1}{\sigma_2}-s+\sigma_1\xi\rho)^{-(m+k)}\Gamma(m+k,(\tfrac{1}{\sigma_2}-s+\sigma_1\xi\rho)\gamma_T)\end{aligned} \tag{20}$$

where

$$\begin{aligned}\mathcal{M}^{**}(s) &= \sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\binom{m+k-1}{i}\frac{\rho^k\xi^{m+k}(-1)^i(1-\rho)^{-k}}{k!\Gamma(m+k)\Gamma(m)\chi^{m+k+i}}\\ &\times\frac{\gamma(m+k+i,\chi\gamma_T)}{(\sigma_1\xi-s)^{m+k-i}}\Big\{\Gamma(m+k-i,\gamma_T(\sigma_1\xi-s))\\ &-\Gamma(m+k-i,2\gamma_T(\sigma_1\xi-s))\Big\},\end{aligned} \tag{21}$$

which can be further simplified by the same techniques shown in (18) and (19) when $\sigma_1=\sigma_2=\sigma$ and/or $\rho=0$.

III. Performance analysis

A. Average combined SNR

The average combined SNR of dual diversity systems is given by

$$\overline{\gamma} = \int_0^{\infty}\gamma\sum_{L_c=1}^{2} f(\gamma,L_c)d\gamma. \tag{22}$$

1) OT-MRC: Substituting (3) into (22) and some mathematical manipulation lead to

$$\begin{aligned}\overline{\gamma}_{\text{OM}} &= \frac{\sigma_1\Gamma(m+1,\frac{\gamma_T}{\sigma_1})}{\Gamma(m)}+\sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\binom{m+k-1}{i}\\ &\times\frac{\rho^k\xi^{i-1}\sigma_1^{i-m-k-1}(-1)^i(1-\rho)^{-k}}{k!\Gamma(m+k)\Gamma(m)\chi^{m+k+i}}\\ &\times\gamma(m+k+i,\chi\gamma_T)\Gamma(m+k-i+1,\gamma_T\sigma\xi)\\ &+\sum_{k=0}^{m-1}\frac{\Gamma(m+k)\xi^m}{k!\Gamma(m)\Gamma(m-k)(2\beta)^{m+k}}\\ &\times\Big\{\frac{(-1)^k}{(\alpha-\beta)^{m-k+1}}\gamma(m-k+1,(\alpha-\beta)\gamma_T)\\ &+\frac{(-1)^m}{(\alpha+\beta)^{m-k+1}}\gamma(m-k+1,(\alpha+\beta)\gamma_T)\Big\}\end{aligned} \tag{23}$$

2) MEC-GSC: Similarly, substituting (12) - (14) into (22) and using the similar manipulation shown previously, we obtain

$$\overline{\gamma}_{\text{MEC}} = \sum_{k=0}^{m-1} \frac{\Gamma(m+k)}{\Gamma(k+1)\Gamma(m)\Gamma(m-k)[\sigma_1\sigma_2(1-\rho)]^m(2\beta)^{m+k}}$$
$$\times \left\{ \frac{(-1)^k}{(\alpha-\beta)^{m-k+1}} \Big(2\gamma(m-k+1,(\alpha-\beta)\gamma_T)$$
$$- \gamma(m-k+1, 2(\alpha-\beta)\gamma_T) \Big)$$
$$+ \frac{(-1)^m}{(\alpha+\beta)^{m-k+1}} \Big(2\gamma(m-k+1,(\alpha+\beta)\gamma_T)$$
$$- \gamma(m-k+1, 2(\alpha+\beta)\gamma_T) \Big) \right\}$$
$$+ \sum_{k=0}^{\infty} \sum_{i=0}^{m+k-1} \binom{m+k-1}{i}$$
$$\times \frac{\rho^k \xi^{i-1} \sigma_1^{i-m-k-1} (-1)^i (1-\rho)^{-k}}{k!\Gamma(m+k)\Gamma(m)\chi^{m+k+i}} \gamma(m+k+i, \chi\gamma_T)$$
$$\times \{ \Gamma(m+k-i+1, \gamma_T\sigma_1\xi) - \Gamma(m+k-i+1, 2\gamma_T\sigma_1\xi) \}$$
$$+ \frac{\sigma_1 \Gamma(m+1, \frac{\gamma_T}{\sigma_1})}{\Gamma(m)} + \frac{\sigma_2 \Gamma(m+1, \frac{\gamma_T}{\sigma_2})}{\Gamma(m)}$$
$$- \frac{e^{\sigma_2\xi\gamma_T}}{\Gamma(m)\sigma_2^m} \sum_{k=0}^{\infty} \sum_{n=0}^{m+k-1} \frac{1}{k!n!} (\sigma_1\xi\rho)^k (\sigma_2\xi\gamma_T)^n$$
$$\times \left(\frac{\sigma_2}{1+\xi\sigma_1\sigma_2\rho} \right)^{m+k+1} \Gamma(m+k+1, (1+\xi\sigma_1\sigma_2\rho)\frac{\gamma_T}{\sigma_2}) \tag{24}$$

B. Average number of branches

The average number of branches is given by

$$\overline{L_c} = \sum_{L_c=1}^{2} L_c \int_0^\infty f(\gamma, L_c) d\gamma. \tag{25}$$

1) OT-MRC: Substituting (3) into (25) and after some mathematical manipulation, we get the average number of branches as

$$\overline{L_c}_{\text{OM}} = \frac{\Gamma(m, \frac{\gamma_T}{\sigma_1})}{\Gamma(m)} + 2 \sum_{k=0}^{m-1} \frac{\Gamma(m+k)\xi^m}{k!\Gamma(m)\Gamma(m-k)(2\beta)^{m+k}}$$
$$\times \left\{ \frac{(-1)^k}{(\alpha-\beta)^{m-k}} \gamma(m-k, (\alpha-\beta)\gamma_T) \right.$$
$$\left. + \frac{(-1)^m}{(\alpha+\beta)^{m-k}} \gamma(m-k, (\alpha+\beta)\gamma_T) \right\} + 2\mathcal{M}^*(0), \tag{26}$$

where $\mathcal{M}^*(s)$ is defined in (17).

2) MEC-GSC: Substituting (12) - (14) into (25) and after some mathematical manipulation, we get the average number of branches as

$$\overline{L_c}_{\text{MEC}} = 2 \sum_{k=0}^{m-1} \frac{\Gamma(m+k)\xi^m}{\Gamma(k+1)\Gamma(m)\Gamma(m-k)(2\beta)^{m+k}}$$
$$\times \left\{ \frac{(-1)^k}{(\alpha-\beta)^{m-k}} \Big(2\gamma(m-k, (\alpha-\beta)\gamma_T)$$
$$- \gamma(m-k, 2(\alpha-\beta)\gamma_T) \Big)$$
$$+ \frac{(-1)^m}{(\alpha+\beta)^{m-k}} \Big(2\gamma(m-k, (\alpha+\beta)\gamma_T)$$
$$- \gamma(m-k, 2(\alpha+\beta)\gamma_T) \Big) \right\} + 4\mathcal{M}^{**}(0)$$
$$+ \frac{\Gamma(m, \frac{\gamma_T}{\sigma_1})}{\Gamma(m)} + \frac{\Gamma(m, \frac{\gamma_T}{\sigma_2})}{\Gamma(m)} - \frac{e^{\sigma_2\xi\gamma_T}}{\Gamma(m)\sigma_2^m} \sum_{k=0}^{\infty} \sum_{n=0}^{m+k-1} \frac{1}{k!n!} (\sigma_1\xi\rho)^k$$
$$\times (\sigma_2\xi\gamma_T)^n \left(\frac{\sigma_2}{1+\xi\sigma_1\sigma_2\rho} \right)^{m+k} \Gamma(m+k, (1+\xi\sigma_1\sigma_2\rho)\frac{\gamma_T}{\sigma_2}) \tag{27}$$

where $\mathcal{M}^{**}(s)$ is defined in (21).

C. Average error rate

The symbol error rate (SER), denoted by $P_S(E)$, for M-PSK is obtained as [10]

$$P_S(E) = \frac{1}{\pi} \int_0^{\frac{M-1}{M}\pi} \mathcal{M}\left(-\frac{\sin^2(\pi/M)}{\sin^2(\phi)} \right) d\phi \tag{28}$$

1) OT-MRC: Substituting (16) into (28), the SER for dual OT-MRC can be derived as

$$P_{S_{\text{OM}}}(E) = \frac{1}{\sigma_1^m} \mathcal{I}_1\left(m, \sigma_1, \frac{\gamma_T}{\sigma_1}\right) + P_S^*(E)$$
$$+ \sum_{k=0}^{m-1} \frac{\Gamma(m+k)\xi^m}{\Gamma(k+1)\Gamma(m)(2\beta)^{m+k}}$$
$$\times \left\{ (-1)^k \mathcal{I}_2\left(m-k, \frac{1}{\alpha-\beta}, \gamma_T(\alpha-\beta)\right) \right.$$
$$\left. + (-1)^m \mathcal{I}_2\left(m-k, \frac{1}{\alpha+\beta}, \gamma_T(\alpha+\beta)\right) \right\}, \tag{29}$$

where

$$\mathcal{I}_1(a,b,c) = \frac{b^a}{\pi} \sum_{i=0}^{a-1} \frac{c^i}{i!} \int_0^{\frac{M-1}{M}\pi} \left(\frac{\sin^2\phi}{\sin^2\phi + b\sin^2(\pi/M)} \right)^{a-i} \times e^{-c\frac{\sin^2\phi + b\sin^2(\pi/M)}{\sin^2\phi}} d\phi,$$
$$\mathcal{I}_2(a,b,c) = \frac{b^a}{\pi} \left\{ \int_0^{\frac{M-1}{M}\pi} \left(\frac{\sin^2\phi}{\sin^2\phi + b\sin^2(\pi/M)} \right)^a d\phi - \sum_{i=0}^{a-1} \frac{c^i}{i!} \int_0^{\frac{M-1}{M}\pi} \left(\frac{\sin^2\phi}{\sin^2\phi + b\sin^2(\pi/M)} \right)^{a-i} \times e^{-c\frac{\sin^2\phi + b\sin^2(\pi/M)}{\sin^2\phi}} d\phi \right\}, \tag{30}$$

and

$$P_S^*(E) = \sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\binom{m+k-1}{i}\frac{\rho^k\xi^{m+k}(-1)^i(1-\rho)^{-k}}{k!\Gamma(m+k)\Gamma(m)\chi^{m+k+i}} \times \Gamma(m+k-i)\gamma(m+k+i,\chi\gamma_T) \times \mathcal{I}_1\left(m+k-i,\tfrac{1}{\sigma_1\xi},\gamma_T\sigma_1\xi\right). \quad (31)$$

If the branch powers are equal, i.e., $\sigma_1=\sigma_2=\sigma$, (35) can be written as

$$P_S^*(E) = \sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\frac{\rho^k\xi^{m+k}\gamma_T^{m+k+i}(-1)^i}{k!i!(m+k+i)\Gamma(m)(1-\rho)^k} \times \mathcal{I}_1\left(m+k-i,\frac{1}{\sigma\xi},\gamma_T\sigma\xi\right), \quad (32)$$

which can be rewritten for $\rho=0$ as

$$P_S^*(E) = \sum_{i=0}^{m-1}\frac{\xi^m\gamma_T^{m+i}(-1)^i}{i!(m+i)\Gamma(m)}\mathcal{I}_1\left(m-i,\frac{1}{\sigma\xi},\gamma_T\sigma\xi\right). \quad (33)$$

2) MEC-GSC: Substituting (20) into (28), the SER for dual MS-GSC can be derived as

$$\begin{aligned}P_{S_{\mathrm{MEC}}}(E) &= \sum_{k=0}^{m-1}\frac{\Gamma(m+k)\xi^m}{\Gamma(k+1)\Gamma(m)(2\beta)^{m+k}} \\ &\times\Bigg\{(-1)^k\Bigg[2\mathcal{I}_2\left(m-k,\tfrac{1}{\alpha-\beta},\gamma_T(\alpha-\beta)\right) \\ &-\mathcal{I}_2\left(m-k,\tfrac{1}{\alpha-\beta},2\gamma_T(\alpha-\beta)\right)\Bigg] \\ &+(-1)^m\Bigg[2\mathcal{I}_2\left(m-k,\tfrac{1}{\alpha+\beta},\gamma_T(\alpha+\beta)\right) \\ &-\mathcal{I}_2\left(m-k,\tfrac{1}{\alpha+\beta},2\gamma_T(\alpha+\beta)\right)\Bigg]\Bigg\} + 2P_S^{**}(E) \\ &+\frac{1}{\sigma_1^m}\mathcal{I}_1\left(m,\sigma_1,\frac{\gamma_T}{\sigma_1}\right) + \frac{1}{\sigma_2^m}\mathcal{I}_1\left(m,\sigma_2,\frac{\gamma_T}{\sigma_2}\right) \\ &-\frac{e^{\sigma_2\xi\gamma_T}}{\Gamma(m)\sigma_2^m}\sum_{k=0}^{\infty}\sum_{n=0}^{m+k-1}\frac{1}{k!n!}(\sigma_1\xi\rho)^k(\sigma_2\xi\gamma_T)^n \\ &\times\Gamma(m+k)\mathcal{I}_1\left(m+k,\tfrac{\sigma_2}{1+\xi\sigma_1\sigma_2\rho},(1+\xi\sigma_1\sigma_2\rho)\tfrac{\gamma_T}{\sigma_2}\right)\end{aligned} \quad (34)$$

where

$$\begin{aligned}P_S^{**}(E) &= \sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\binom{m+k-1}{i}\frac{\rho^k\xi^{m+k}(-1)^i(1-\rho)^{-k}}{k!\Gamma(m+k)\Gamma(m)\chi^{m+k+i}} \\ &\times\Gamma(m+k-i)\gamma(m+k+i,\chi\gamma_T) \\ &\times\Bigg\{\mathcal{I}_1\left(m+k-i,\frac{1}{\sigma_1\xi},\gamma_T\sigma_1\xi\right) \\ &-\mathcal{I}_1\left(m+k-i,\frac{1}{\sigma_1\xi},2\gamma_T\sigma_1\xi\right)\Bigg\}.\end{aligned} \quad (35)$$

If the branch powers are equal, i.e., $\sigma_1=\sigma_2=\sigma$, (34) can be written as

$$\begin{aligned}P_{S_{\mathrm{MEC}}}(E) &= \sum_{k=0}^{m-1}\frac{\Gamma(m+k)\xi^m}{\Gamma(k+1)\Gamma(m)(2\beta)^{m+k}} \\ &\times\Bigg\{(-1)^k\Bigg[2\mathcal{I}_2\left(m-k,\frac{1}{\alpha-\beta},\gamma_T(\alpha-\beta)\right) \\ &-\mathcal{I}_2\left(m-k,\frac{1}{\alpha-\beta},2\gamma_T(\alpha-\beta)\right)\Bigg] \\ &+(-1)^m\Bigg[2\mathcal{I}_2\left(m-k,\frac{1}{\alpha+\beta},\gamma_T(\alpha+\beta)\right) \\ &-\mathcal{I}_2\left(m-k,\frac{1}{\alpha+\beta},2\gamma_T(\alpha+\beta)\right)\Bigg]\Bigg\} \\ &+2\sum_{k=0}^{\infty}\sum_{i=0}^{m+k-1}\frac{\rho^k\xi^{m+k}\gamma_T^{m+k+i}(-1)^i}{k!i!(m+k+i)\Gamma(m)(1-\rho)^k} \\ &\times\Bigg\{\mathcal{I}_1\left(m+k-i,\frac{1}{\sigma\xi},\gamma_T\sigma\xi\right)-\mathcal{I}_1\left(m+k-i,\frac{1}{\sigma\xi},2\gamma_T\sigma\xi\right)\Bigg\} \\ &+\frac{1}{\sigma^m}\mathcal{I}_1\left(m,\sigma,\frac{\gamma_T}{\sigma}\right)-\frac{e^{\sigma\xi\gamma_T}}{\Gamma(m)\sigma^m}\sum_{k=0}^{\infty}\sum_{n=0}^{m+k-1}\frac{1}{k!n!}(\sigma\xi\rho)^k(\sigma\xi\gamma_T)^n \\ &\times\Gamma(m+k)\mathcal{I}_1\left(m+k,\frac{\sigma}{1+\xi\sigma^2\rho},(1+\xi\sigma^2\rho)\frac{\gamma_T}{\sigma}\right)\end{aligned} \quad (36)$$

IV. NUMERICAL RESULTS

As numerical examples we consider the nonidentical distributions with a linear relation between two SNRs of diversity paths such as $\overline{\gamma}_2 = q\overline{\gamma}_1$ and set $q=0.9$. The target SNR and the Nakagami parameter are also fixed as $\gamma_T = 7$ dB and $m=2$, respectively.

Fig. 1 shows the average combined SNR for dual OT-MRC and MEC-GSC, when $\rho = 0$, 0.3, 0.6, and 0.9, and compared with the performance of no diversity (single branch) and dual MRC. We can see that, when the average branch SNR is zero dB, the average combined SNR of both OT-MRC and MEC-GSC follows the combined SNR of dual MRC with $\rho=0$, and it gradually steps onto the SNR of a single branch (i.e. no diversity) as the average SNR per branch increases. Fig.2 shows the ratio of average combined SNR of dual OT-MRC and MEC-GSC, when $\rho = 0$, 0.3, 0.6, and 0.9. As shown in this figure, OT-MRC scheme slightly outperforms MEC-GSC around preselected threshold by $0 \sim 0.45$dB depending on the degree of correlation.

Fig. 3 shows the average number of branches for dual OT-MRC and MEC-GSC when $\rho = 0$, 0.3, 0.6, and 0.9. First note that the average number of combined branches of OT-MRC is not affected by the correlation coefficient. Also note that for any value of the correlation coefficient the MEC-GSC requires smaller average number of combined branches than the OT-MRC, while the difference of BER performance between OT-MRC and MEC-GSC is negligible over the correlation coefficient as shown in Fig. 4.

Therefore we can conclude that the OT-MRC is more efficient in terms of the power-consumption, while satisfying the required target threshold.

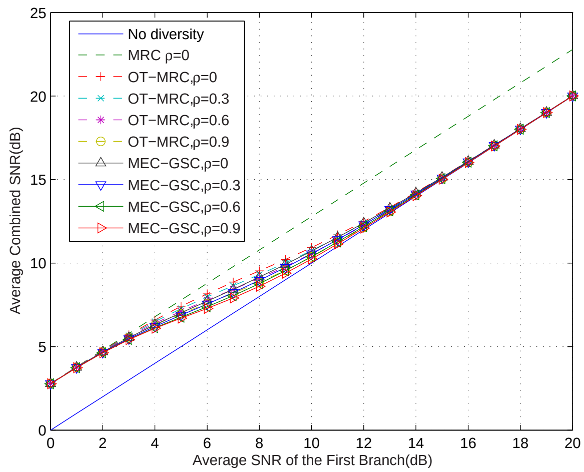

Fig. 1. Average combined SNR of dual OT-MRC and MEC-GSC over correlated Nakagami-m fading channels for $\gamma_T = 7$ dB

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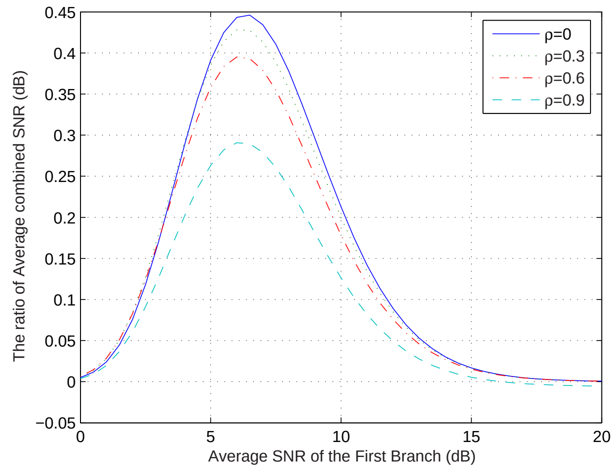

Fig. 2. The ratio of Average combined SNR of dual OT-MRC and MEC-GSC over correlated Nakagami-m fading channels for $\gamma_T = 7$ dB

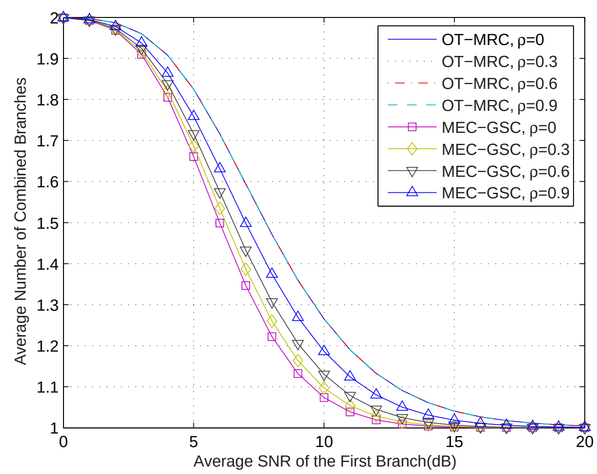

Fig. 3. Average number of selected branches of dual OT-MRC and MEC-GSC over correlated Nakagami-m fading channels for $\gamma_T = 7$ dB

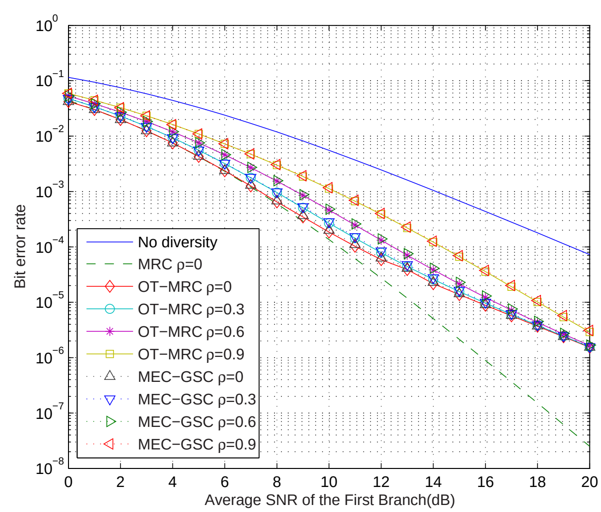

Fig. 4. Bit error rate of dual OT-MRC and MS-GSC over correlated Nakagami-m fading channels for $\gamma_T = 7$ dB